Calculating the gauge-invariant two-point correlation function of gluonic field strengths

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ABSTRACT: The two-point gauge-invariant correlation function of gluonic field strengths, which is the main input in the stochastic vacuum model, is derived by using its relation to the Green functions of one- and two-gluon gluelumps. These Green functions are found by evaluating the path integrals of one and two gluons confined in the field of an infinitely heavy adjoint source. This allows one to obtain also the terms in the exponential asymptotics of the two-point correlation function, which are produced by fluctuations of the gluons around their mean separation from the source. The pre-exponents are derived as well, demonstrating some novel dependences on the distance.

KEYWORDS: Phenomenological Models, Lattice Gauge Field Theories, QCD, Nonperturbative Effects.

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Contents

1. Introduction

Despite a remarkable impact of the stochastic vacuum model [1, 2] on QCD phenomenology (see e.g. [3] for a recent review), a field-theoretic establishment of this model is still pending. The main problem is an analytic calculation of the nonperturbative content of the two-point gauge-invariant correlation function of gluonic field strengths, which can be parametrized as [2]

$$\frac{g^2}{N_c} \text{tr} \left\langle F_{\mu\nu}(x)\Phi_{xx'} F_{\lambda\rho}(x')\Phi_{x'x} \right\rangle_{\text{YM}} = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})D(x - x') + \tag{1.1}$$

$$+ \frac{1}{2} \left[\partial_\mu^x \left((x - x')_\lambda \delta_{\nu\rho} - (x - x')_\rho \delta_{\nu\lambda}\right) + \partial_\nu^x \left((x - x')_\rho \delta_{\mu\lambda} - (x - x')_\lambda \delta_{\mu\rho}\right) \right] D_1(x - x').$$

Here, $\Phi_{xx'} \equiv \mathcal{P} \exp\left(ig \int_{x'}^{x} dz_\mu A_\mu^a(z) T_{\text{fund}}^a\right)$ is a phase factor along the straight line, which goes through x' and x; T_{fund}^a's stand for the generators of the group $\text{SU}(N_c)$ in the fundamental representation; the average $\langle\ldots\rangle_{\text{YM}}$ is taken with respect to the Euclidean Yang-Mills action, $\frac{1}{4}\int d^4x (F_{\mu\nu}^a)^2$; $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$, $a = 1, \ldots, N_c^2 - 1$.

Numerous lattice measurements of the correlation function (1.1) were performing over the years (see [1, 4] for the calculations with the cooling method and ref. [5], where this correlation function was reconstructed from the lattice data on the static quark-antiquark potential). As for the papers attempting to analytically evaluate this quantity, some of those were dealing with the Abelian theories, where confinement was produced by monopoles [6, 7]. At the same time, some progress was achieving in the non-Abelian case as well [8, 9] (see also ref. [10] for the calculations of the correlation function (1.1) within the instanton vacuum model). In particular, a relation between the correlation length of the vacuum (that is the length at which the functions D and D_1 experience an exponential fall-off) and the gluonic condensate was derived in [8]. Further analysis of the correlation lengths of D and D_1 was done in [9].[1] Recently, a complete derivation of these functions

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[1]It was supporting the conjecture, tested on the lattice in [5], that these lengths might be different.

was presented in [11]. Using initially the idea on a relation between the correlation function (1.1) and the Green functions of one and two (valence) gluons confined in the field of an infinitely heavy adjoint source (the so-called one- and two-gluon gluelumps), the paper [11] further accommodated the Hamiltonian formalism for the analysis of these Green functions. This eventually yielded certain nonperturbative expressions for the functions D and D_1.

In the present paper, the Green functions (as well as the functions D and D_1) will be calculated upon an evaluation of the corresponding integrals over trajectories of the gluons in the gluelumps. It turns out that, when the minimal-area-law Ansatz is applied to the Wilson loops of these (confined) gluons, further calculation of the path integrals becomes possible by making use of the Feynman variational method. By means of this method, we manage to find some novel terms in both the exponents and pre-exponents of the functions D and D_1, which stem from the vibrations of the gluons around their mean separations from the heavy source.

The paper is organized as follows. In section 2, we demonstrate our method at the example of a one-gluon gluelump and calculate in this way the function D_1. In section 3, we proceed with the two-gluon gluelump and the function D. In section 4, a brief summary of the obtained results is presented.

2. Green function of a one-gluon gluelump and the function D_1

In the approximation when the gluon spin term is disregarded [11], the one-gluon gluelump Green function has the form $\delta_{\mu\nu}G(T,0)$, where

$$G(T,0) = \int\limits_0^\infty ds \int\limits_{x_4(0)=0,\, x_4(s)=T} \mathcal{D}x_\mu \exp\left(-\int_0^s d\tau \frac{\dot{x}_\mu^2}{4}\right) \langle W(C)\rangle_{\mathrm{YM}}.$$

Here, in the Wilson loop $\langle W(C)\rangle_{\mathrm{YM}} = \mathrm{tr}\left\langle \mathcal{P}\exp\left(ig\oint_C dx_\mu A_\mu^a T_{\mathrm{adj}}^a\right)\right\rangle_{\mathrm{YM}}$, T_{adj}^a's stand for the $SU(N_c)$-generators in the adjoint representation, $(T_{\mathrm{adj}}^a)^{bc} = -if^{abc}$; the closed contour C is formed by the piece of the 4-th axis from the point $(\mathbf{0},T)$ to the origin and the trajectory $x_\mu(\tau)$ of a gluon, which moves back from the origin to that point. Adopting for $\langle W(C)\rangle_{\mathrm{YM}}$ the minimal-area-law Ansatz, we have

$$G(T,0) = N_c(N_c^2-1)\int\limits_0^\infty ds \int\limits_{x_4(0)=0,\, x_4(s)=T} \mathcal{D}x_4 \int \mathcal{D}\mathbf{x} \times$$
$$\times \exp\left[-\int_0^s d\tau \left(\frac{\dot{\mathbf{x}}^2}{4} + \frac{\dot{x}_4^2}{4} + \sigma_{\mathrm{adj}}|\mathbf{x}|\dot{x}_4\right)\right].$$

It will further be assumed that, in the directions transverse to x_4, gluon, along most of its path, only slightly fluctuates around some mean distance, L, from the 4-th axis. [2] This

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[2]This distance will be found numerically at the end of the calculation.

means the following parametrization $\mathbf{x}(\tau) = \mathbf{L} + \mathbf{l}(\tau)$, where $\mathbf{l}(0) = \mathbf{l}(s) = 0$, $\mathbf{L} = L\mathbf{n}$. Then, up to the term linear in $\mathbf{l}$, $|\mathbf{x}| \simeq L + \mathbf{nl}$. In this approximation, we have

$$
G(T,0) \simeq N_c(N_c^2 - 1)\mathrm{e}^{-\sigma_{\mathrm{adj}}LT} \int\limits_0^\infty ds \int\limits_{x_4(0)=0,\, x_4(s)=T} \mathcal{D}x_4 \int \mathcal{D}\mathbf{l}\mathcal{D}\mu \times
$$

$$
\times \exp\left[-\int_0^s d\tau \left(\frac{\dot{\mathbf{l}}^2}{4} + \frac{\dot{x}_4^2}{4} + \frac{\mu}{2} + \frac{\sigma_{\mathrm{adj}}^2 \dot{x}_4^2 (\mathbf{nl})^2}{2\mu} \right) \right],
$$

where we have introduced an integration over the "einbein" field μ in order to make the potential term quadratic in $\mathbf{l}$. Next, the unit vector $\mathbf{n}$ may, without loss of generality, be chosen along the 1-st axis, $\mathbf{n} = (1, 0, 0)$. The integrations over $l_2(\tau)$ and $l_3(\tau)$ then become those of a free particle. As for the integration over $l_1(\tau)$, since $\mathbf{l}$, by definition, only weakly varies along the trajectory, one may replace $l_1^2(\tau)$ by its value averaged along the trajectory, $l_1^2 \to \frac{1}{s}\int_0^s d\tau l_1^2$. Altogether we have

$$
G(T,0) \simeq N_c(N_c^2 - 1)\mathrm{e}^{-\sigma_{\mathrm{adj}}LT} \int\limits_0^\infty \frac{ds}{4\pi s} \int\limits_{x_4(0)=0,\, x_4(s)=T} \mathcal{D}x_4 \int \mathcal{D}l_1\mathcal{D}\mu \times
$$

$$
\times \exp\left[-\int_0^s d\tau \left(\frac{\dot{l}_1^2}{4} + \frac{\dot{x}_4^2}{4} + \frac{\mu}{2} + U(l_1) \right) \right], \quad U(l_1) = \left(\frac{\sigma_{\mathrm{adj}}^2}{2s} \int_0^s d\tau \frac{\dot{x}_4^2}{\mu} \right) l_1^2. \quad (2.1)
$$

Although the partition function of a harmonic oscillator, resulting now from the integration over $l_1(\tau)$, is known exactly, its substitution would be of little use for a practical calculation. (Merely since the frequency of this oscillator is a functional of $\dot{x}_4/\mu$, and the subsequent integrals over x_4 and μ can hardly be handled.) Rather, it is much more useful to apply Feynman variational method [12], that is fully legitimate, since l_1 only slightly fluctuates around the origin. This method yields the leading quantum correction to the classical expression

$$
\frac{1}{\sqrt{4\pi s}}\mathrm{e}^{-sU(0)} \quad (2.2)
$$

for the integral $\int\limits_{x(0)=x(s)=0} \mathcal{D}x(\tau) \exp\left[-\int_0^s d\tau \left(\frac{\dot{x}^2}{4} + U(x) \right) \right]$. The correction originates just from the fluctuations of the trajectory around the origin and leads to the following expression for the above-mentioned integral (see e.g. [13] for the details of derivation): $\frac{1}{\sqrt{4\pi s}} \exp\left[-\sqrt{\frac{3s}{\pi}} \int_{-\infty}^{+\infty} dx U(x)\mathrm{e}^{-\frac{3x^2}{s}} \right]$. The classical approximation (2.2) is recoverable from this formula in the limit $s \to 0$.

Substituting for U the expression from eq. (2.1), we then get for the path integral over $l_1(\tau)$: $\frac{1}{\sqrt{4\pi s}} \exp\left(-\frac{\sigma_{\mathrm{adj}}^2 s}{12} \int_0^s d\tau \frac{\dot{x}_4^2}{\mu} \right)$. This yields

$$
G(T,0) = N_c(N_c^2 - 1)\mathrm{e}^{-\sigma_{\mathrm{adj}}LT} \int\limits_0^\infty \frac{ds}{(4\pi s)^{3/2}} \int\limits_{x_4(0)=0,\, x_4(s)=T} \mathcal{D}x_4\mathcal{D}\mu \times
$$

$$\times \exp\left[-\int_0^s d\tau \left(\frac{\dot{x}_4^2}{4} + \frac{\mu}{2} + \frac{\sigma_{\rm adj}^2 s}{12}\frac{\dot{x}_4^2}{\mu}\right)\right]. \tag{2.3}$$

Further integration over μ replaces the sum $\frac{\mu}{2} + \frac{\sigma_{\rm adj}^2 s}{12}\frac{\dot{x}_4^2}{\mu}$ by a single term $\sigma_{\rm adj}\sqrt{s/6}|\dot{x}_4|$. This is a world-line Lagrangian of a free scalar particle, which moves along the 4-th axis. (The mass of this particle $m \equiv \sigma_{\rm adj}\sqrt{s/6}$ is, however, a function of s.) Such a Lagrangian is known to be equivalent to the one usually used for the world-line calculations, $\frac{\dot{x}_4^2}{4} + m^2$, so that the total world-line action reads $\int_0^s d\tau \frac{\dot{x}_4^2}{2} + m^2 s$. To restore the standard coefficient $1/4$ at the kinetic term, one can perform the rescaling $\tau = 2\tau_{\rm new}$, $s = 2s_{\rm new}$. The integral over x_4 then becomes standard, and we obtain

$$G(T,0) = N_c(N_c^2 - 1)\frac{\mathrm{e}^{-\sigma_{\rm adj}LT}}{16\sqrt{2}\pi^2}\int\limits_0^\infty \frac{ds}{s^2}\exp\left(-\frac{T^2}{4s} - \frac{2\sigma_{\rm adj}^2}{3}s^2\right).$$

To calculate the integral over s, let us change the integration variable to $\xi = (\sigma s)^{-1}$, that yields for this integral $\sigma \int_0^\infty d\xi \exp\left(-\frac{\sigma_{\rm adj}T^2}{4}\xi - \frac{2}{3\xi^2}\right)$. The saddle-point approximation to the integral $\int_0^\infty d\xi \mathrm{e}^{-f(\xi)}$ works well when the width of the corresponding Gaussian distribution, which is $\frac{1}{\sqrt{f''(\xi_0)}}$ (where ξ_0 is the saddle point), is much smaller than 1. In our case, $\frac{1}{\sqrt{f''(\xi_0)}} = (32/9)^{1/3}(\sigma_{\rm adj}T^2)^{-2/3}$, therefore the saddle-point approximation is legitimate when the distances under consideration are large in the sense of the inequality $\sigma_{\rm adj}T^2 \gg 1$. Calculation of the integral is then straightforward, and we have

$$G(T,0) = N_c(N_c^2 - 1)\frac{(\sigma_{\rm adj}/T^4)^{1/3}}{3^{2/3}\cdot 2^{7/3}\cdot\pi^{3/2}}\exp\left[-\sigma_{\rm adj}LT - (9/32)^{1/3}(\sigma_{\rm adj}T^2)^{2/3}\right].$$

The function $D_1(x)$ is related to this expression as [11] $D_1(x) = -\frac{2g^2}{N_c^2}\frac{dG(x,0)}{dx^2}$. Performing the differentiation we obtain

$$D_1(x) = \frac{C_2\alpha_s}{\sqrt{\pi}}(32/9)^{1/3}\frac{\sigma_{\rm adj}}{x^2}\left[\frac{1}{12^{1/3}} + \frac{L}{2}\left(\frac{\sigma_{\rm adj}}{|x|}\right)^{1/3} + \frac{2}{3(\sigma_{\rm adj}x^2)^{2/3}}\right]\times$$

$$\times \exp\left[-\sigma_{\rm adj}L|x| - (9/32)^{1/3}(\sigma_{\rm adj}x^2)^{2/3}\right] \text{ at } \sigma_{\rm adj}x^2 \gg 1, \tag{2.4}$$

where $C_2 = \frac{N_c^2 - 1}{2N_c}$ is the quadratic Casimir operator of the fundamental representation. Note that a gluon is confined only at large N_c, where $\sigma_{\rm adj} = 2\sigma_{\rm fund}$ (while, at $N_c \sim 1$, it is screened by other gluons). In the large-N_c limit, $C_2\alpha_s \to \frac{\lambda}{8\pi}$, where $\lambda = g^2 N_c$ is the so-called 't Hooft coupling, which stays finite in this limit.

Equation (2.4) can further be compared to the expression for $D_1(x)$ found in ref. [11] by the Hamiltonian approach, which reads

$$D_1(x) = \frac{2C_2\alpha_s M_0\sigma_{\rm adj}}{|x|}\mathrm{e}^{-M_0|x|} \text{ valid at } M_0|x| \gg 1, \tag{2.5}$$

where $M_0 \simeq 1.5\,\mathrm{GeV}$ at $\sigma_{\rm fund} = 0.18\,\mathrm{GeV}^2$. One can see that the term $(9/32)^{1/3}(\sigma_{\rm adj}x^2)^{2/3}$ in eq. (2.4), although parametrically dominating the exponent in the limit $\sigma_{\rm adj}x^2 \gg 1$,

is numerically smaller than the leading term $\sigma_{\rm adj}L|x|$. This is not surprising, since the term $(9/32)^{1/3}(\sigma_{\rm adj}x^2)^{2/3}$, originating from small fluctuations of the valence gluon around its mean separation from the heavy source, can only be a correction to the leading result. Indeed, equating $\sigma_{\rm adj}L$ to M_0, we have $\frac{(9/32)^{1/3}(\sigma_{\rm adj}x^2)^{2/3}}{\sigma_{\rm adj}L|x|} = \frac{\left(9\sigma_{\rm fund}^2|x|\right)^{1/3}}{2M_0}$. Next, in the pure Yang-Mills theory which we consider here, the stochastic vacuum model is only applicable up to the distances where the glueball production starts. This happens at $|x|_{\rm max} = \frac{M_{\rm glueball}}{\sigma_{\rm fund}}$. Setting for an estimate $M_{\rm glueball} = 1\,{\rm GeV}$, we get $|x|_{\rm max} = 5.56\,{\rm GeV}^{-1}$. (This yields $\sigma_{\rm adj}|x|_{\rm max}^2 = 11.11$, that still leaves lot of space for the inequality $\sigma_{\rm adj}x^2 \gg 1$ to hold.) Accordingly, $\frac{\left(9\sigma_{\rm fund}^2|x|\right)^{1/3}}{2M_0} \le 0.39$, that demonstrates the numerical smallness of the term $(9/32)^{1/3}(\sigma_{\rm adj}x^2)^{2/3}$ with respect to the term $\sigma_{\rm adj}L|x|$.

Let us finally consider the pre-exponent of eq. (2.4). First, the relation $\sigma_{\rm adj}L = M_0$ yields $L = 0.92\,{\rm fm}$. This value for the mean distance, by which a valence gluon is separated from an infinitely heavy adjoint source, well exceeds the corresponding vacuum correlation length $M_0^{-1} \simeq 0.15\,{\rm fm}$. The string in a gluelump is therefore a clearly shaped object. One can further see that, at this value of L, the term $\frac{1}{12^{1/3}}$ in the brackets is always larger than the term $\frac{L}{2}\left(\frac{\sigma_{\rm adj}}{|x|}\right)^{1/3}$. Indeed, to compare the latter to the former means to compare $\frac{\sigma_{\rm adj}L^3}{|x|} = \frac{26.08\,{\rm GeV}^{-1}}{|x|}$ to $2/3$ (at $|x| \gg \frac{1}{\sqrt{\sigma_{\rm adj}}} = 1.67\,{\rm GeV}^{-1}$). Clearly, $\frac{26.08\,{\rm GeV}^{-1}}{|x|} < \frac{2}{3}$ up to $|x| = 39.12\,{\rm GeV}^{-1}$, that is the distance much larger than $|x|_{\rm max}$. This proves our statement that $\frac{1}{12^{1/3}} > \frac{L}{2}\left(\frac{\sigma_{\rm adj}}{|x|}\right)^{1/3}$. As for the term $\frac{2}{3(\sigma_{\rm adj}x^2)^{2/3}}$, it exceeds $\frac{1}{12^{1/3}}$ only in the limiting case $\sigma_{\rm adj}x^2 < 1.89$, that is on the border of applicability of the inequality $\sigma_{\rm adj}x^2 \gg 1$. Thus, we conclude that, in general, the term $\frac{1}{12^{1/3}}$ is the defining one for the pre-exponential behavior of the function $D_1(x)$. This behavior is, therefore, $\sim x^{-2}$, that differs from the behavior $\sim |x|^{-1}$ of eq. (2.5).

3. Green function of a two-gluon gluelump and the function D

This gluelump is formed by two gluons, which are attached by fundamental strings to an infinitely heavy adjoint source and joined to each other by a third fundamental string. Since the gluons are identical, the vectors of their spatial coordinates, $\mathbf{x}(\tau)$ and $\mathbf{y}(\tau)$, can be chosen in the form $\mathbf{x}(\tau) = L'\mathbf{n}_1 + \mathbf{l}_1(\tau)$, $\mathbf{y}(\tau) = L'\mathbf{n}_2 + \mathbf{l}_2(\tau)$. Here, $\mathbf{n}_1 = (\cos\theta, -\sin\theta, 0)$, $\mathbf{n}_2 = (\cos\theta, \sin\theta, 0)$, and θ is some constant angle, whose possible values as a function of L' will be discussed at the end of the calculation. The mean distance L' from the gluons to the heavy source is, in general, different from L. Within such notations, the area swept out by the string joining the gluons is $2L'T\sin\theta$. After the extraction of the mean-area dependent factor, the scalar function $G^{(2{\rm gl})}(T, 0)$, through which the Green function of the gluelump is expressed [11], factorizes into the product of integrals over trajectories of each of the gluons:

$$G^{(2{\rm gl})}(T, 0) = {\rm e}^{-2\sigma_{\rm fund}L'T\sin\theta} \int\limits_0^\infty ds \int\limits_0^\infty ds' \int\limits_{x_4(0)=0,\, x_4(s)=T} \mathcal{D}x_4 \int\limits_{y_4(0)=0,\, y_4(s)=T} \mathcal{D}y_4 \int \mathcal{D}\mathbf{x}\mathcal{D}\mathbf{y} \times$$

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$$\times \exp\left[-\int_0^s d\tau \left(\frac{\dot{\mathbf{x}}^2}{4} + \frac{\dot{x}_4^2}{4} + \sigma_{\text{fund}}|\mathbf{x}|\dot{x}_4\right) - \int_0^{s'} d\tau \left(\frac{\dot{\mathbf{y}}^2}{4} + \frac{\dot{y}_4^2}{4} + \sigma_{\text{fund}}|\mathbf{y}|\dot{y}_4\right)\right].$$

(3.1)

Let us consider, for example, the integral over $\mathbf{x}(\tau)$. Substituting $|\mathbf{x}| \simeq L' + \mathbf{n}_1\mathbf{l}_1$, we have for it

$$e^{-\sigma_{\text{fund}}L'T} \int \mathcal{D}\mathbf{l}_1 \mathcal{D}\mu \exp\left[-\int_0^s d\tau \left(\frac{\dot{\mathbf{l}}_1^2}{4} + \frac{\mu}{2} + \frac{\sigma_{\text{fund}}^2 \dot{x}_4^2 (\mathbf{n}_1\mathbf{l}_1)^2}{2\mu}\right)\right].$$

We can further use the same approximation as in the previous section, namely substitute $(\mathbf{n}_1\mathbf{l}_1)^2 \to \frac{1}{s}\int_0^s d\tau (\mathbf{n}_1\mathbf{l}_1)^2$. Together with the (free) integration over $l_1^3(\tau)$, this yields

$$\frac{e^{-\sigma_{\text{fund}}L'T}}{\sqrt{4\pi s}} \int \mathcal{D}\mathcal{R}\mathcal{D}\mu \exp\left[-\int_0^s d\tau \left(\frac{\dot{\mathcal{R}}^2}{4} + \frac{\mu}{2} + U(\mathcal{R})\right)\right],$$

where $\mathcal{R} \equiv (l_1^1, l_1^2)$ and

$$U(\mathcal{R}) = \left(\frac{\sigma_{\text{fund}}^2}{2s}\int_0^s d\tau \frac{\dot{x}_4^2}{\mu}\right)(l_1^1\cos\theta - l_1^2\sin\theta)^2.$$

Using the variational method in the 2-d case (see [13] for details), we further obtain

$$\int \mathcal{D}\mathcal{R} \exp\left[-\int_0^s d\tau \left(\frac{\dot{\mathcal{R}}^2}{4} + U(\mathcal{R})\right)\right] \simeq \frac{1}{4\pi s}\exp\left[-\frac{3}{\pi}\int d^2\mathcal{R} U(\mathcal{R})e^{-\frac{3\mathcal{R}^2}{s}}\right] =$$

$$= \frac{1}{4\pi s}\exp\left(-\frac{\sigma_{\text{fund}}^2 s}{12}\int_0^s d\tau \frac{\dot{x}_4^2}{\mu}\right).$$

The resulting integral over x_4 and μ then coincides (modulo the replacement $\sigma_{\text{adj}} \to \sigma_{\text{fund}}$) with that of eq. (2.3), so that the total integral over $ds\mathcal{D}x_4\mathcal{D}\mathbf{x}$ in eq. (3.1) reads

$$\frac{(\sigma_{\text{fund}}/T^4)^{1/3}}{3^{2/3}\cdot 2^{7/3}\cdot\pi^{3/2}}\exp\left[-\sigma_{\text{fund}}L'T - (9/32)^{1/3}(\sigma_{\text{fund}}T^2)^{2/3}\right].$$

The integral over $ds'\mathcal{D}y_4\mathcal{D}\mathbf{y}$ apparently produces the same result. Taking into account the relation [11] $D(x) = \frac{g^4(N_c^2-1)}{2}G^{(2\text{gl})}(x,0)$, we finally obtain that, at $\sigma_{\text{fund}}x^2 \gg 1$,

$$D(x) = \frac{g^4(N_c^2-1)}{3^{4/3}\cdot 2^{17/3}\cdot\pi^3}\left(\frac{\sigma_{\text{fund}}}{|x|^4}\right)^{2/3}\exp\left[-2\sigma_{\text{fund}}L'(1+\sin\theta)|x| - (9/4)^{1/3}(\sigma_{\text{fund}}x^2)^{2/3}\right].$$

(3.2)

Again, as in the case of the function $D_1(x)$, the term $2\sigma_{\text{fund}}L'(1+\sin\theta)|x|$ in the exponent is numerically larger than $(9/4)^{1/3}(\sigma_{\text{fund}}x^2)^{2/3}$. Indeed, identifying $2\sigma_{\text{fund}}L'(1+\sin\theta)$ with $M_0^{(2\text{gl})} = 2.56\,\text{GeV}$ from the formula [11]

$$D(x) \simeq \frac{g^4(N_c^2-1)}{2}0.108\sigma_{\text{fund}}^2 e^{-M_0^{(2\text{gl})}|x|} \text{ valid at } M_0^{(2\text{gl})}|x| \gg 1,$$

(3.3)

we have $\frac{(9/4)^{1/3}(\sigma_{\text{fund}}x^2)^{2/3}}{2\sigma_{\text{fund}}L'(1+\sin\theta)|x|} \leq \frac{\left(9\sigma_{\text{fund}}^2|x|_{\text{max}}/4\right)^{1/3}}{M_0^{(2\text{gl})}} = 0.12$. Furthermore, the mean distance between the two gluons, as well as the mean distance between these gluons and the heavy

source may not exceed $|x|_{\max}$, i.e. the inequalities $2L' \sin \theta \leq |x|_{\max}$, $L' \leq |x|_{\max}$ should be respected together with the equation $2\sigma_{\text{fund}} L'(1 + \sin \theta) = M_0^{(2\text{gl})}$. One can readily see that the unphysical case $\theta = 0$ cannot be realized also numerically. Indeed, one should then have $\frac{M_0^{(2\text{gl})}}{\sigma_{\text{fund}}} = 2L' \leq 2|x|_{\max}$, that is not feasible, since the l.h.s. equals $14.22 \, \text{GeV}^{-1}$, whereas the r.h.s. equals $11.12 \, \text{GeV}^{-1}$. For all other cases, we have: $\theta \leq \arcsin \frac{|x|_{\max}}{2L'}$ at $\frac{|x|_{\max}}{2} \leq L' \leq |x|_{\max}$, while no restrictions on the values of θ exist at $L' < \frac{|x|_{\max}}{2}$. [3] Finally, we note that the pre-exponent in the function (3.2) falls off as $|x|^{-8/3}$, whereas the pre-exponent of the function (3.3) is a constant.

4. Summary

In the present paper, we have evaluated the functions $D(x)$ and $D_1(x)$, which parametrize the two-point gauge-invariant correlation function of gluonic field strengths in the stochastic vacuum model of QCD. To this end, we have applied the Feynman variational method to the calculation of the Green functions of one- and two-gluon gluelumps, through which the functions D and D_1 can be expressed. At the end of the calculation, we have compared our results with those found by the Hamiltonian analysis in ref. [11]. In case of the function D_1, this comparison yielded the mean separation of a gluon from the heavy source in the one-gluon gluelump. In case of the function D, it yielded some constraints on the values of the angle between the planar world sheets of the gluons in the two-gluon gluelump. We have further seen that, although fluctuations of the gluons (which are accounted for by the variational method) around their mean separations from the source produce the terms in the exponential asymptotics of the functions D and D_1, which are parametrically larger than the leading ones, numerically these terms are nevertheless subleading, as they should be. The gluon fluctuations yield also the distance dependences of the pre-exponential factors in the both functions D and D_1, which are different from those of ref. [11], where these fluctuations were not considered. In fact, the effective string Hamiltonian employed in ref. [11] was not taking into account vibrations of the end-point of the string. Rather, those are described within the Feynman variational method used in the present paper. The differences in the pre-exponents of the functions D and D_1 are, however, of a rather academic interest, since numerically each of the pre-exponents is subdominant with respect to the corresponding exponent at large distances considered both here and in ref. [11].

In conclusion, the obtained results might be important for a field-theoretic establishment of the stochastic vacuum model. Besides that, the performed calculation demonstrates how one can account for the fact that the virtual particles in QCD diagrams are confined rather than free. Last but not least, the novel expressions for D and D_1 obtained here might be useful for applications in high-energy scattering.

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[3]Note that L' is bounded from below only by the inequality $L' > \frac{1}{M_0^{(2\text{gl})}}$.

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